

$$P_i = (R, \downarrow \text{lon}_i, \pi/2 - \text{lat}_i) \quad \text{spherical coords}$$

$$P_i = (R \sin(\text{colat}_i) \cos(\text{lon}_i), R \sin(\text{colat}_i) \sin(\text{lon}_i), R \cos(\text{colat}_i)) \quad \text{rectangular coords}$$

$$\vec{a} \cdot \vec{b} = |a| |b| \cos(\alpha)$$

$$\begin{aligned} \cos(\alpha) &= \cos(\text{lat}_1) \cos(\text{lon}_1) \cdot \cos(\text{lat}_2) \cos(\text{lon}_2) \\ &\quad + \cos(\text{lat}_1) \sin(\text{lon}_1) \cos(\text{lat}_2) \sin(\text{lon}_2) \\ &\quad + \sin(\text{lat}_1) \sin(\text{lat}_2) \\ &= \cos(\text{lat}_1) \cos(\text{lat}_2) (\cos(\text{lon}_1) \cos(\text{lon}_2) \\ &\quad + \sin(\text{lon}_1) \sin(\text{lon}_2)) \\ &\quad + \sin(\text{lat}_1) \sin(\text{lat}_2) \\ &= \cos(\text{lat}_1) \cos(\text{lat}_2) \cos(\text{lon}_1 - \text{lon}_2) + \sin(\text{lat}_1) \sin(\text{lat}_2) \\ &= \cos^{-1} [\cos(\text{lat}_1) \cos(\text{lat}_2) \cos(\text{lon}_1 - \text{lon}_2) + \sin(\text{lat}_1) \sin(\text{lat}_2)] \end{aligned}$$

$d$  = great circle distance between points

$$= R \alpha$$

, losing  $\frac{b^w}{(1-l)}$  growth per bet, is  $G$ , where  
 $G = \lim_{N \rightarrow \infty} \frac{1}{N} \ln(V_N/V_0)$   
 $V_N$  wins and  $V_0$  losses

after  $\left( \frac{1-l}{1+l} \right)^w \left( \frac{1-l}{1+l} \right)^L$

$$G = \lim_{N \rightarrow \infty} \left[ \frac{W}{N} \ln \left( \frac{1+lb}{1+l} \right) + \frac{L}{N} \ln \left( \frac{1-lb}{1-l} \right) \right] \xrightarrow{W \rightarrow P} 1-P$$

In limit,  $\frac{W}{N} \rightarrow P$  and  $L \rightarrow 1-P$

$$G = P \ln \left( \frac{1+lb}{1+l} \right) + (1-P) \ln \left( \frac{1-lb}{1-l} \right)$$

Find  $l$  which maximizes  $G$ . Call it  $l_{\text{opt}}$

$$\frac{dG}{dl}_{l=l_{\text{opt}}} = 0 = \frac{Pb}{1+l_{\text{opt}}+b} - \frac{1-P}{1-l_{\text{opt}}} \Rightarrow l_{\text{opt}} = \frac{\sqrt{P(b+1)} - 1}{b} = \frac{Pb - q}{b}$$

where  $q = \sqrt{P(b+1)} - 1$

or  $l_{\text{opt}} = \frac{iP_{\text{track}}}{1+iP_{\text{track}}} = \frac{P - P_{\text{track}}}{1 - P_{\text{track}}}$

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$$\frac{l_{\text{opt}}}{1} = \frac{iP_{\text{track}}}{1+iP_{\text{track}}} \quad \text{or} \quad \frac{l_{\text{opt}}}{1} = \frac{P - P_{\text{track}}}{1 - P_{\text{track}}} \quad \text{term}$$